Multiple scattering model for pion-nucleon scattering and the slope of the Pomeranchuk trajectory

1972 J. Phys. A: Gen. Phys. 5 838

(http://iopscience.iop.org/0022-3689/5/6/009)

J. Phys. A: Gen. Phys., Vol. 5, June 1972. Printed in Great Britain

Multiple scattering model for pion–nucleon scattering and the slope of the Pomeranchuk trajectory

D FINCHAM

Department of Natural Philosophy, University of Aberdeen, Aberdeen AB9 2UE, UK

MS received 3 November 1971, in revised form 3 January 1972

Abstract. The multiple scattering model of Frautschi and Margolis, with secondary trajectories included, is applied to pion–nucleon elastic and charge-exchange scattering. It is possible to explain the Serpukhov high energy total cross section data, but not other data such as the phase of the forward amplitude and the elastic scattering polarization.

1. Introduction

A possible explanation for the levelling off in the downward trend of pion–nucleon total cross sections above 30 GeV/c incident momentum is the presence of Regge cuts in the scattering amplitude (Barger and Phillips 1970). One method of producing such cuts is by applying absorption to the Regge pole exchange amplitudes. This technique has been applied by several authors to pion–nucleon scattering, the most exhaustive analysis being that of Carerras and White (1970), who give references to previous work. It is found that the ρ pomeron cut generated by the absorption, while it produces the cross over in the $\pi^{\pm}$p elastic differential cross sections, and the polarization in the charge-exchange reaction, is not sufficient to explain the high energy total cross sections. This leads us to consider cuts generated by multiple pomeron exchanges.

This idea was first used by Frautschi and Margolis (1968) to discuss proton–proton elastic scattering. The main features of such a model (the FM model) are:

(i) The pomeron trajectory has nonzero slope and the differential cross section shrinks indefinitely with increasing energy. Other authors (eg Chiu and Finkelstein 1968, Arnold and Blackmon 1968) have considered similar models but with a purely imaginary amplitude equivalent to a zero slope pomeron. These seem to be ruled out by the Serpukhov data (Beznogikh *et al* 1969) which show that the pp elastic differential cross section is still shrinking at 70 GeV/c.

(ii) The cuts produced by successively higher orders of scattering dominate the differential cross section at successively higher values of momentum transfer. The different terms interfere to give dips in the differential cross section such as are observed in pp scattering.

(iii) The model produces the correlation between large total cross section and narrow forward peak width expected on a simple diffractive picture of the elastic scattering.

(iv) The total cross section (σ_{tot}) rises to its asymptotic value. A positive value for β, the ratio of real to imaginary parts of the forward amplitude, is produced. (We call this quantity β, rather than the more standard α, since α is also commonly used for trajectory functions.)

The falling σ_{tot} and negative β at laboratory energies must be explained by the presence of secondary trajectories, not considered by Frautschi and Margolis. Kaplan and Schiff (1970) included secondary trajectories and applied the resulting model (which we will call the FMS model) to pp and $\bar{p}p$ elastic scattering. They were able to fit most features of the data *except* the total cross section, because in the Serpukhov data σ_{tot} for $\bar{p}p$ does not flatten off, but continues to fall steadily with increasing energy.

The present work extends the FMS model to πN elastic and charge exchange scattering. While it was in progress the results of Hamer and Ravndal (1970) became available. These authors carry out similar calculations for a range of reactions which include πN scattering. The main differences from the present work are that they do not take spin into account and their discussion is restricted to elastic scattering. It will be seen later that our conclusions differ somewhat from theirs.

The details of the model are given in § 2 of the paper. In § 3 we discuss the trajectories of the Regge poles with particular reference to the pomeron trajectory. The results of the calculations are given in § 4 and conclusions drawn in § 5.

2. The model

Our model is one of a class of models where the scattering amplitude T is obtained from a 'single scattering' term B by the prescription

$$\begin{aligned} T &= -i\{\exp(iB)-1\} \\ &= B+\frac{i}{2!}B^2-\frac{1}{3!}B^3+\ldots. \end{aligned} \tag{1}$$

Such an expression is familiar from the theory of nonrelativistic scattering as the eikonal approximation, if B is interpreted as the Born term. The obvious relativistic generalization is to replace the Born term by the Feynman diagram corresponding to elementary particle exchange, but such a model takes no account of the presence of inelastic channels, and an alternative is to use Regge pole exchange for B. The terms in higher powers of B then have the form of Regge cuts. The theoretical justification for this 'Regge pole eikonal model' has been discussed recently by Cicuta and Sugar (1971), and in the present paper we restrict ourselves to the study of some of its phenomenological consequences.

The equation (1) was written somewhat formally. In the strict eikonal model it applies in impact parameter space. The scattering amplitude is then found as a function of momentum transfer by a two dimensional Fourier transform. Instead, we choose to apply equation (1) to the partial wave amplitudes. The term in B^2 for example then represents two successive scatters with the intermediate particles on the mass shell. From the point of view of the phenomenology it makes no difference which method is adopted. For example, we repeated the calculation of Hamer and Ravndal using an identical Born term, but calculating the full amplitude with the aid of the partial wave multiple scattering series rather than using the impact parameter method. The results agreed to within better than one per cent.

Our technique is thus the following. We start off with Regge pole amplitudes which are functions of energy and of the centre of mass scattering angle θ. Since we wish to take spin into account, there are two amplitudes, corresponding to helicity nonflip,

$B_{++}(\theta)$, and to helicity flip, $B_{+-}(\theta)$. We then carry out the partial wave decompositions

$$B^J_{++} = \tfrac{1}{2}\int_{-1}^{1} B_{++}(\theta) d^J_{\frac{1}{2}\frac{1}{2}}(\theta)\, \mathrm{d}\cos\theta$$

$$B^J_{+-} = \tfrac{1}{2}\int_{-1}^{1} B_{+-}(\theta) \mathrm{d}^J_{-\frac{1}{2}\frac{1}{2}}(\theta)\, \mathrm{d}\cos\theta \tag{2}$$

and construct the matrix

$$B^J = \begin{bmatrix} B^J_{++} & B^J_{+-} \\ B^J_{+-} & B^J_{++} \end{bmatrix}.$$

This is then substituted in the multiple scattering series, equation (1), to give T^J, the matrix of the full partial wave helicity amplitudes. The partial wave series is then resummed to give the final amplitudes:

$$T_1 = T_{++}(\theta) = \sum_{J=1/2}^{\infty} (2J+1) T^J_{++} d^J_{\frac{1}{2}\frac{1}{2}}(\theta)$$

$$T_2 = T_{+-}(\theta) = \sum_{J=1/2}^{\infty} (2J+1) T^J_{+-} d^J_{-\frac{1}{2}\frac{1}{2}}(\theta). \tag{3}$$

In terms of these amplitudes the experimental quantities are

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\pi}{k^2}(|T_1|^2 + |T_2|^2)$$

$$\sigma_{\mathrm{tot}} = \frac{4\pi}{k} \operatorname{Im} T_1(\theta)$$

$$P = \frac{2 \operatorname{Im}(T_1 T_2^*)}{|T_1|^2 + |T_2|^2}. \tag{4}$$

Here P is the polarization, and k is the momentum in the centre of mass frame.

In pion–nucleon scattering there are three relevant Regge pole exchanges, the pomeron, the f and the ρ. Thus we have for the elastic scattering

$$B(\pi^+\mathrm{p}) = B_{\mathrm{p}} + B_{\mathrm{f}} - B_{\rho}$$

$$B(\pi^-\mathrm{p}) = B_{\mathrm{p}} + B_{\mathrm{f}} + B_{\rho}. \tag{5}$$

Once the elastic scattering amplitudes have been calculated, the amplitude for the charge-exchange reaction $\pi^-\mathrm{p} \to \pi^0\mathrm{n}$ is given by

$$T(\mathrm{cex}) = \frac{T(\pi^+\mathrm{p}) - T(\pi^-\mathrm{p})}{\sqrt{2}}. \tag{6}$$

This gives a model similar, but not identical, to the absorption model. For example, expanding to second order equation (6) gives

$$T(\mathrm{cex}) = \sqrt{2}\{B_{\rho} + \mathrm{i}(B_{\mathrm{p}} + B_{\mathrm{f}})B_{\rho} + \ldots\}.$$

The absorption model on the other hand gives

$$T(\mathrm{cex}) = B_{\rho}(\mathrm{cex}) + \mathrm{i}B_{\rho}(\mathrm{cex})T(\pi^-\mathrm{p})$$
$$= \sqrt{2}\{B_{\rho} + \mathrm{i}(B_{\mathrm{p}} + B_{\mathrm{f}} + B_{\rho})B_{\rho} + \ldots\}.$$

Thus the absorption model has a ρρ cut whereas our model does not. We note in passing that this cut has positive signature and should be forbidden (Branson 1969). Again, from the point of view of the phenomenology these distinctions make little difference and we expect results similar to the absorption model.

It remains for us to specify the Regge pole amplitudes. We wish to compare our results with those of Arnold and Blackmon (1968) who performed an eikonal model calculation for πN scattering, but using a purely imaginary 'pomeron'. We therefore follow these authors in using a nonsense-choosing rho amplitude, and setting the p and f s channel helicity-flip amplitudes to zero. The amplitudes we employed were

$$\begin{aligned}
B_{\mathrm{p}++} &= g_{\mathrm{p}} \exp(-\mathrm{i}\pi\alpha_{\mathrm{p}}/2)(s/s_0)^{\alpha_{\mathrm{p}}} s_0/\sqrt{s} \\
B_{\mathrm{f}++} &= g_{\mathrm{f}}(\alpha_{\mathrm{f}}+1)\exp(ct)\exp(-\mathrm{i}\pi\alpha_{\mathrm{f}}/2)s^{\alpha_{\mathrm{f}}}/\sqrt{s} \\
B_{\rho++} &= g_{\rho}\alpha_{\rho}(\alpha_{\rho}+2)\exp(at)\mathrm{i}\exp(-\mathrm{i}\pi\alpha_{\rho}/2)s^{\alpha_{\rho}}/\sqrt{s} \\
B_{\rho+-} &= h(-t)^{1/2}\alpha_{\rho}(\alpha_{\rho}+2)\exp(bt)\mathrm{i}\exp(-\mathrm{i}\pi\alpha_{\rho}/2)s^{\alpha_{\rho}}/\sqrt{s}. \qquad (7)
\end{aligned}$$

In these amplitudes, s and t are the usual Mandelstam variables. We have a total of eight parameters; $g_{\mathrm{p}}, s_0, g_{\mathrm{f}}, c, g_{\rho}, a, h_{\rho}$ and b, not counting the trajectory functions. The factors of α_{ρ} ensure that both B_{++} and B_{+-} vanish at the wrong signature point $\alpha_{\rho} = 0$ (the nonsense-choosing mechanism). Unlike Arnold and Blackmon we have for the sake of completeness included factors to make the appropriate amplitudes vanish at the wrong signature points $\alpha_{\mathrm{f}} = -1$ and $\alpha_{\rho} = -2$; these factors have only a small effect in the region of t of interest. Another, more important, difference from their work is that we do not use exchange degeneracy to relate the ρ and f residues; this is because we will require a larger proportion of f exchange to overcome the rising σ_{tot} produced by the multiple exchanges of our pomeron with nonzero slope.

3. Trajectory parameters

We attempted to specify the trajectory parameters in advance, before trying to fit the model to the πN scattering data. Our motivation was to reduce the number of parameters involved in the fitting in order to simplify the work and, more importantly, to bring in as much outside information as possible.

3.1. The pomeron

The best place for studying the pomeron is the reaction $\phi\mathrm{p} \to \phi\mathrm{p}$ since there should be no secondary trajectories contributing (Barger and Cline 1970). This is related by the assumption of vector dominance of electromagnetic interactions to the reaction $\gamma\mathrm{p} \propto \phi\mathrm{p}$, which is accessible experimentally.

Writing the pomeron trajectory as $\alpha_{\mathrm{p}}(t) = 1 + \alpha' t$ where the slope parameter α' is to be determined, a pure pole model gives for the differential cross section for $\gamma\mathrm{p} \to \phi\mathrm{p}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = A\exp\{(b + 2\alpha' \ln s)t\}.$$

We fitted the above expression to the data of Anderson *et al* (1970) and obtained $\alpha' = 0{\cdot}61 \pm 0{\cdot}12$ with $b = 0{\cdot}09 \pm 0{\cdot}78$.

However, we are more interested in the multiple scattering FM model. In this model the spin-independent amplitude with only the pomeron contributing is

$$f(s,t) = \mathrm{i}g \sum_{n=1}^{\infty} \frac{1}{nn!}\left(\frac{-g}{2\alpha'\mu}\right)^{n-1} \exp\left(\frac{t\alpha'\mu}{n}\right) \tag{8}$$

where $\mu = \ln(s/s_0) - \mathrm{i}\pi/2$, and g, s_0 and α' are parameters. Weare (1970) studied the application of this expression to the reaction $\phi p \to \phi p$, and obtained a fit with $\alpha' = 0{\cdot}54$. However, in this work the parameter s_0 was arbitrarily set equal to 1·0. Since there is no t dependence in the residue, this means that the value of α' obtained by the fitting procedure is constrained by the need to obtain the correct slope for the differential cross section, rather than just by its energy dependence. Accordingly we repeated the work of Weare, but allowed s_0 to be a free parameter. This is equivalent to inserting an exponential in t in the amplitude.

The fitting technique was the following. We fit simultaneously to σ_{tot} and to $F(t)$, the ratio of the differential cross section at t to its value at $t = 0$. The fitted points were at a selection of energies between 6 and 20 GeV/c and a selection of values of $|t|$ between 0 and 0·5 (GeV/c)^2. The 'experimental' values of $\sigma_{\text{tot}}(\phi p)$ were the quark model predictions calculated by Weare. To obtain 'experimental' values of $F(t)$ we assumed that it had the same form as in the reaction $\gamma p \to \phi p$, and accordingly generated the points from the expression

$$F(t) = \exp\{(0{\cdot}09 + 1{\cdot}22 \ln s)t\}$$

previously obtained as a best fit to the $\gamma p \to \phi p$ data. To these experimental points we fit the theoretical expressions for σ_{tot} and $F(t)$ obtained from equation (8) for the FM amplitude. The resulting best fit gave $\alpha' = 0{\cdot}59$ with $g = 3{\cdot}11$ and $s_0 = 1{\cdot}16$. The uncertainty in α' is, because of the technique used, approximately equal to the uncertainty in the value obtained in the pole-only fit, namely $\pm 0{\cdot}12$.

The pomeron can also be studied using the data on the forward differential cross section for pp scattering at Serpukhov energies, where presumably the secondary trajectories are not very important. A pole-only fit to these data gives $\alpha' = 0{\cdot}47 \pm 0{\cdot}09$ (Beznogikh *et al* 1969). Kolbig and Margolis (1970) studied them using the FM model and obtained $\alpha' = 1{\cdot}0$. Again in this work s_0 was given an arbitrary value rather than being treated as a parameter, and the discrepancy between this value of α' and that obtained by Weare reflects simply the different values for the differential cross section slope in the two processes. We therefore applied the FM model to high energy pp scattering, using a similar technique to that used in the ϕp reaction. We fit simultaneously to $\sigma_{\text{tot}}(pp)$ using the experimental data of Foley *et al* (1967a) and Galbraith *et al* (1965) between 12 and 26 GeV/c and to $F(t)$ using the expression

$$F(t) = \exp\{(6{\cdot}8 + 0{\cdot}94 \ln s)t\}$$

obtained from the Serpukhov data between 12 and 70 GeV/c. This gave the results $\alpha' = 0{\cdot}45$ with $g = 10{\cdot}2$ and $s_0 = 0{\cdot}0024$. The uncertainty in α' is taken as $\pm 0{\cdot}09$. The values of the parameters are only slightly sensitive to the range of values of $|t|$ employed.

In the two applications of the FM model above, the values of α' obtained are very close to the values given by the pomeron alone without its multiple scattering corrections. This can be understood by reference to figure 1 which shows the effective value of $\alpha(t)$ in the FM model. This graph was obtained by calculating the differential cross section for

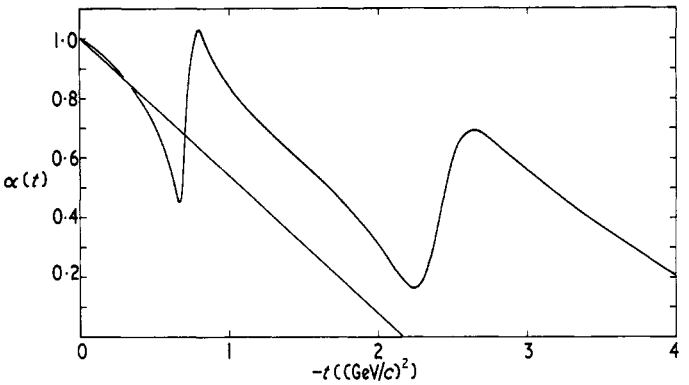

Figure 1. Effective value of $\alpha(t)$ in the Frautschi–Margolis model for pp scattering. The values of the parameters are $g = 10{\cdot}2$, $s_0 = 0{\cdot}0024$, $\alpha' = 0{\cdot}46$. The straight line is α for the pole that is the line $\alpha(t) = 1{\cdot}0 + 0{\cdot}46t$.

pp scattering from equation (8) at several energies between 20 and 70 GeV/c, and fitting to an expression of the form

$$\frac{d\sigma}{dt} = A(t)s^{2\alpha(t)-2}.$$

Both in the ϕp and the pp case we restricted ourselves to small momentum transfers, and it can be seen from figure 1 that in this region the value of $\alpha(t)$ is very close to that given by the pomeron pole alone. The two begin to diverge at larger momentum transfers, where the effective value of $\alpha(t)$ for the FM model has a minimum. This reflects the fact that the differential cross section has a dip structure, which is produced by interference between single and double scattering etc, and this structure sharpens up with increasing energy.

The original application of the FM model was in fact to the study of the structure in the pp elastic differential cross section. In their paper Frautschi and Margolis (1968) obtained $\alpha' = 0{\cdot}82$, with s_0 fixed at 1·0. In a repeat of the calculation we obtained $\alpha' = 0{\cdot}55 \pm 0{\cdot}07$, with $g = 6{\cdot}2$ and $s_0 = 0{\cdot}27$. It should be pointed out that the resulting fit to the data is by no means excellent, as can be seen from figure 2.

The most reliable of the above three estimates for α' is that from the reaction $\gamma p \to \phi p$, since it is not complicated by the presence of secondary trajectories, but it is encouraging to note that the other two estimates are consistent with this. Accordingly in our study of πN scattering we fixed the value of α' at 0·6 $(\text{GeV}/c)^{-2}$.

This value is somewhat larger than most other estimates. Regge pole fits to all the elastic data pre-Serpukhov gave $\alpha' \simeq 0{\cdot}1$–$0{\cdot}3$ (Rarita *et al* 1968). The absorption model calculations of Carreras and White (1970) have $\alpha' \simeq 0{\cdot}3$.

3.2. The ρ *and* f

To fix these trajectories we assume that trajectories occur in exchange degenerate pairs, so that the ρ trajectory must pass through the ρ and A_2 masses whilst the f trajectory must pass through the f and ω masses. These conditions give

$$\alpha_\rho(t) = 0{\cdot}47 + 0{\cdot}90t$$

$$\alpha_f(t) = 0{\cdot}38 + 1{\cdot}01t.$$

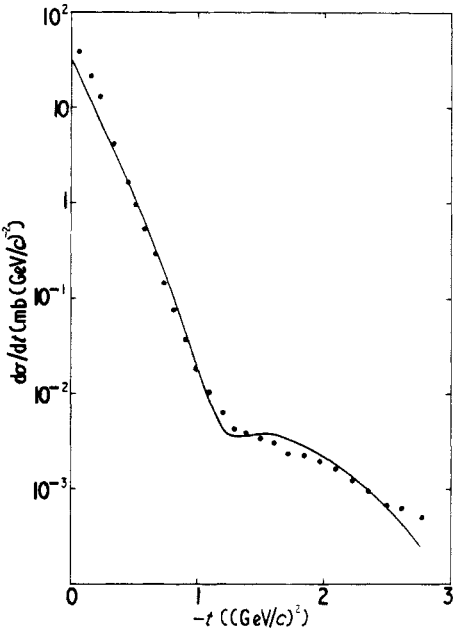

Figure 2. Fit to the proton–proton elastic differential cross section using the Frautschi–Margolis model. The result shown is at 12 GeV/c, and is compared with the data of Allaby *et al* (1968). Data at a variety of energies were used in the fitting process. The quoted error on all the experimental points is $\pm 4\%$.

4. Results and discussion

The fit to the πN scattering data was carried out without the aid of minimization routines, as the main aim was to investigate qualitative features. As will be explained below, it was not possible to obtain a good fit to all the data simultaneously, but we show in table 1 a typical set of parameters on which our discussion will be based. (We call this solution I.)

Table 1. Values of the parameters for solution I

g_p	s_0	g_f	c	g_ρ	a	h_ρ	b
−2·7	0·1	−2·5	2·3	−0·85	−1·0	2·8	0·5

4.1. Total cross sections, phase of forward amplitude, elastic differential cross sections

The major aim of our investigation was to determine whether the model can explain the Serpukhov cross section data. As explained in § 1, the pomeron alone with its multiple scattering corrections gives a rising σ_{tot} and a positive β. We need to adjust the relative magnitudes of p and f exchanges to give falling σ_{tot} and negative β at laboratory energies. Adjustment of the ρ residue gives the difference between the π^+p and π^-p values. Figure 3 shows the total cross sections as functions of energy, and it can be seen

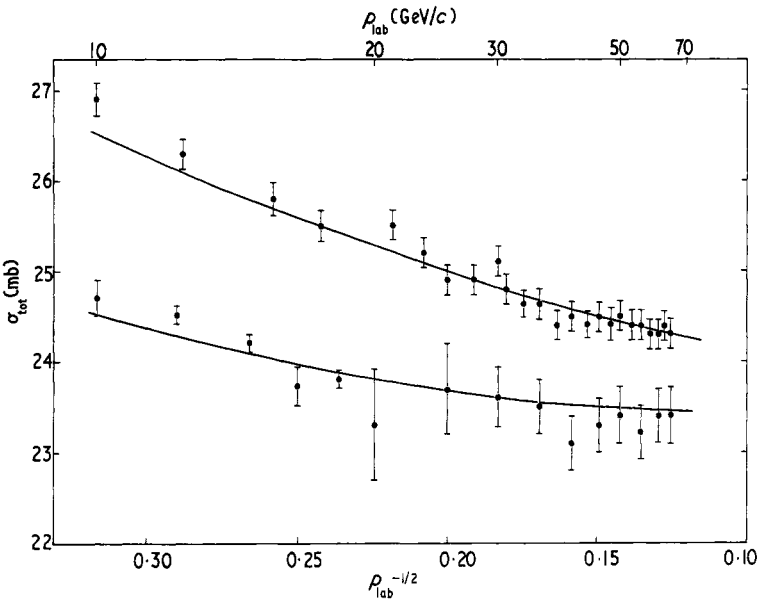

Figure 3. Total cross sections for π^+p and π^-p scattering. The data are from Foley *et al* (1967b), Galbraith *et al* (1965), Allaby *et al* (1969) and Vasiljev (1970 unpublished).

that a good fit to the data is obtained. Our large value of α' is a help in obtaining this agreement, since the pomeron and its iterations then give a rapidly increasing σ_{tot}. This combines with the falling secondary terms to reproduce the change in slope of the graphs. We were not able to obtain quite such a good fit to the data when using a value of 0·45 for α'_p.

In figure 4 we show the values of β for solution I, and it is clear that the theoretical values are not nearly negative enough. They can be made more negative by increasing

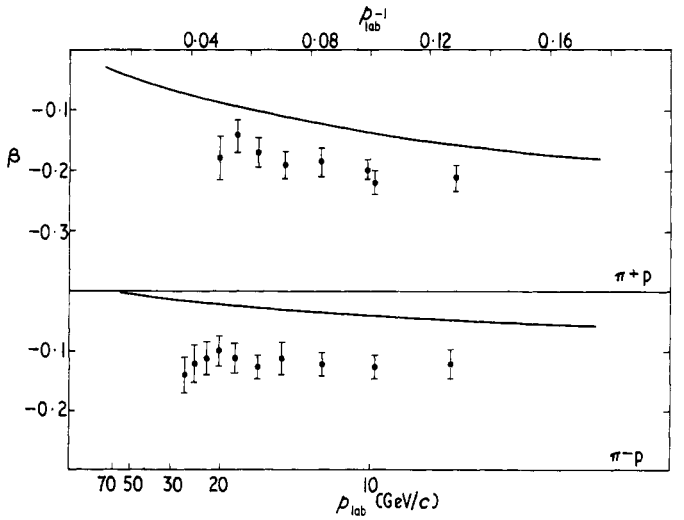

Figure 4. Ratio of real to imaginary parts of the forward scattering amplitude. The data are from Foley *et al* (1969).

the proportion of f exchange compared with p exchange (already much larger than in the Regge pole model), but this then destroys the agreement with the σ_{tot} data. It is impossible to fit σ_{tot} and β simultaneously.

The solution of Hamer and Ravndal (1970) is of the second type, with a larger proportion of f exchange giving a good fit to β but not to σ_{tot}. (In a note added in proof to their paper, these authors suggest that their solution may be in agreement with the revised Serpukhov data (Vasiljev 1970) but in this they were over optimistic.) We may hope to distinguish between the two types of solution by studying the elastic differential cross sections, as their energy dependence will be different in the two cases. In figure 5 we show the results of solution I, and in figure 6 of a calculation (which we call solution II) in which the p and f amplitudes are those used by Hamer and Ravndal, and to which we have added a ρ flip term identical to I. Solution I gives a distinctly better set of results than solution II. (We comment on the cross-over effect in these data in § 4.2.) However, this is hardly a fair way of differentiating between the two solutions, as we have not attempted to vary the parameters of Hamer and Ravndal to improve the fit. It is perhaps more illuminating to investigate the qualitative features of the two types of solution. The pomeron term alone, because of its nonzero slope, gives a rapidly shrinking forward peak in the differential cross sections. An f contribution which, as in the present examples, is more sharply peaked than the p, tends to counteract this effect as it decreases rapidly

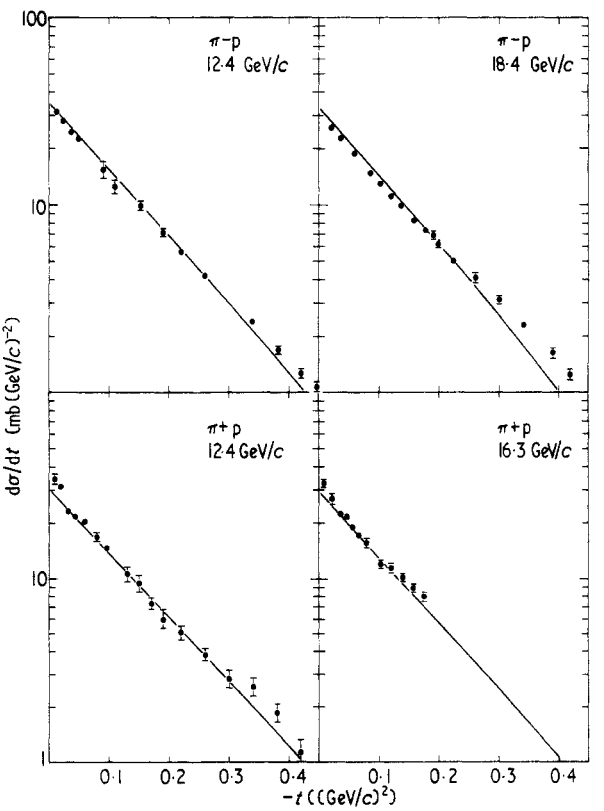

Figure 5. Elastic scattering differential cross sections. The data are from Harting *et al* (1965), and Foley *et al* (1969). In this and subsequent figures error bars are omitted where they are too small to be shown clearly.

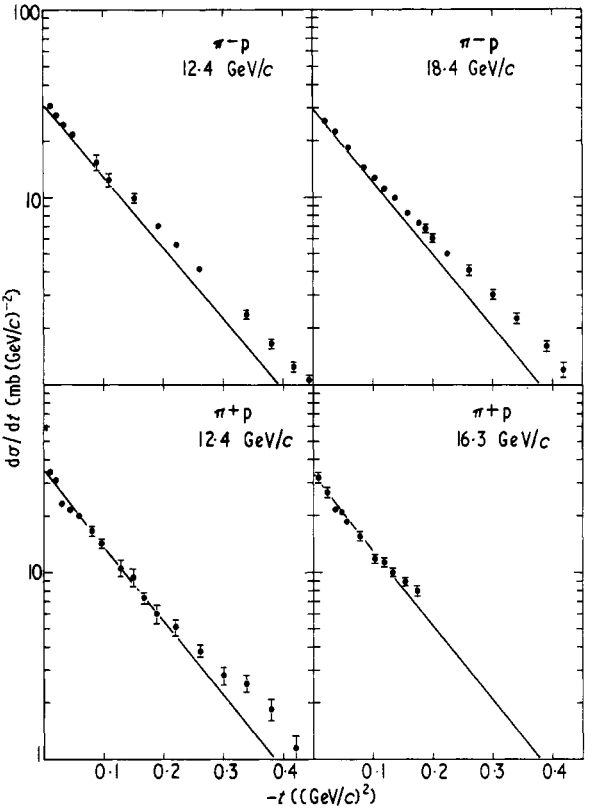

Figure 6. Elastic scattering differential cross sections for solution II.

in magnitude with energy. (This is discussed in detail in Rarita *et al* 1968.) Thus solution II with its larger f contribution shows much less shrinkage than solution I. In their work Hamer and Ravndal fitted to forward slopes calculated from the data of Foley *et al* (1963), which do indeed show little shrinkage. However, the later and much more detailed data of Foley *et al* (1969) do show considerable shrinkage, and a solution of type I seems to be preferred. We do not, though, regard this argument as conclusive since the rate of shrinkage depends on the slope of the pomeron trajectory. The arguments of § 3 were only suggestive, and until we can investigate the pomeron singularity more accurately, for example by experiments at very high energies, the question must be regarded as still open. Another way of saying the same thing is to note that the relative proportions of p and f exchanges determine the rate of change of the rate of shrinkage with energy, and the present data are not yet good enough to make decisive statements about this quantity.

4.2. Charge exchange scattering and elastic scattering polarizations

The residue of the ρ nonflip amplitude is largely determined by the total cross section difference. We then have to vary h_ρ, a and b to fit the charge exchange differential cross section. We found that it was not possible to obtain a very good fit, and in figure 7 we show the results of solution I. With these parameters we have a good fit in the forward

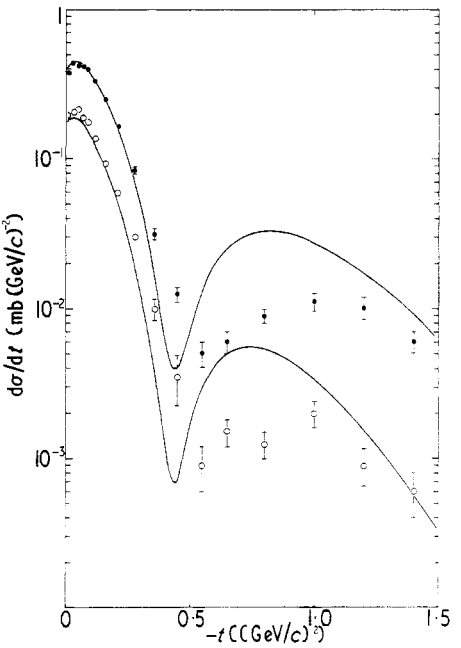

Figure 7. Results for the charge exchange differential cross section at 5·9 GeV/c (upper curve) and 13·3 GeV/c. The data are from Stirling *et al* (1965), and Sonderegger *et al* (1966).

direction, but the secondary peak is a factor of two too large. It is possible to adjust the parameters to reduce its size, but then the positions of the dip and secondary peak move to smaller values of $|t|$, spoiling the fit in the forward direction. We can understand this poor fit by reference to White (1969) and Carreras and White (1970) who study the absorption model for this reaction. They show that for a good fit the quantity $N = \nu B/A'$ for (p+f) exchanges must be negative, a value of $N = -2$ giving best results. The amplitudes A' and B are related to the s channel helicity amplitudes in such a way that setting the flip amplitudes to zero, as in our work, is equivalent to fixing N at a value of $+1$. Our results could be improved by introducing appropriate p and f flip amplitudes to give the required negative value. A positive N has the effect of making the cuts too large; it is well known that the ρ Regge pole alone gives a good fit to the charge exchange differential cross section. However, it should be pointed out that finite energy sum rules predict a positive value for N (Barger and Phillips 1968). This suggests that the absorption model is not giving good results for the actual scattering amplitudes, since it can only fit the data well with a negative N.

For the purpose of comparison we have carried out a fit using the ρ pole alone, taking the amplitudes from equation (7). The resulting fit is shown in figure 8. Its χ^2 is not very good, for two reasons: our ρ is nonsense choosing and so has a zero rather than a dip at $\alpha_\rho(t) = 0$; and the trajectory was constrained as explained in § 3.2. If we allow the slope and intercept to be free parameters, we find a much better fit with $\alpha(t) = 0{\cdot}62 + 1{\cdot}12t$.

Introducing p and flip amplitudes would also help with another problem: the cross-over effect. Experimentally the π^+p and π^-p elastic differential cross sections cross over somewhere between $|t| = 0{\cdot}1$ and $|t| = 0{\cdot}3$. Our cuts do give the effect, but in solution I

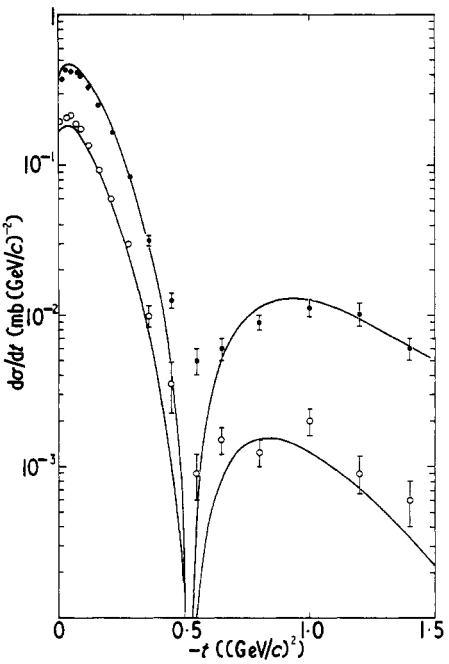

Figure 8. Fit to charge exchange differential cross section at 5·9 and 13·3 GeV/c using ρ pole alone. The parameters are $g = 0{\cdot}85$, $h = -3{\cdot}5$, $a = -0{\cdot}79$, $b = 0{\cdot}83$.

the cross-over point is at $|t| = 0{\cdot}46$, and the best that can be obtained by varying parameters is $|t| = 0{\cdot}34$.

Our result for the elastic scattering polarizations is shown in figure 9. It will be seen that the theoretical curves, unlike the data, change sign near $|t| = 0{\cdot}4$. This is to be contrasted with the results of Arnold and Blackmon (1968) who obtained a good fit in

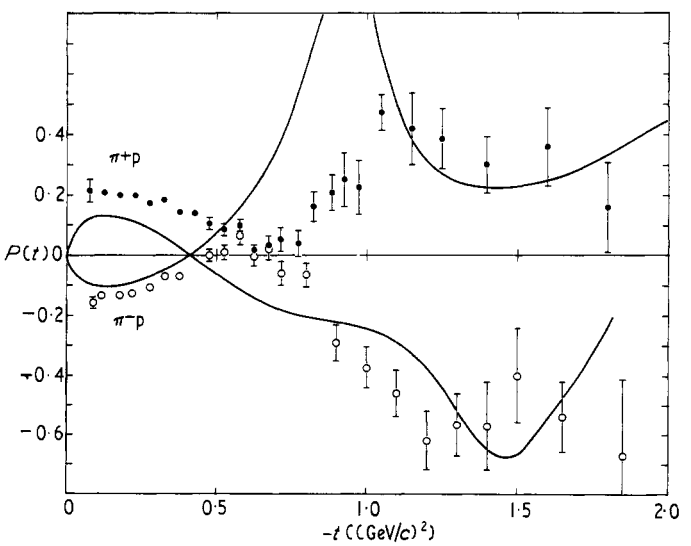

Figure 9. Elastic scattering polarizations at 6 GeV/c. The data are from Borghini *et al* (1970).

a Regge eikonal model with a purely imaginary 'pomeron' amplitude. The difference between the results can be understood as follows. Since the ρ exchange is largely helicity flip the polarization (equation (4)) becomes

$$P \propto \mathrm{Im}(V_{++}\rho^*_{+-})$$
$$= \mathrm{Im}\, V_{++}\, \mathrm{Re}\, \rho_{+-} - \mathrm{Re}\, V_{++}\, \mathrm{Im}\, \rho_{+-}.$$

Here V stands for the sum of p and f exchanges. In Arnold and Blackmon only the first term is important because the f contribution is small and the p is entirely imaginary. $\mathrm{Re}\,\rho_{+-}$ has a double zero at $\alpha_\rho = 0$, and so the polarization stays on the same side of the axis. In our work, the pomeron amplitude rapidly develops a large real part as $|t|$ increases, because of the large slope of its trajectory, and the second term quickly dominates. Now at $\alpha_\rho = 0$, $\mathrm{Im}\,\rho_{+-}$ has only a single zero, and so a change of sign is produced. We investigated the effect of including an extra factor of α in the f amplitude (the no-compensation mechanism). Since the point $\alpha_f = 0$ is close to $\alpha_\rho = 0$ this has the effect of introducing a double zero in the term $\mathrm{Re}\, f_{++}\ \mathrm{Im}\ \rho_{+-}$. The results were changed only slightly from those of figure 9, showing that it is indeed the large real part to the pomeron amplitude which is the cause of the trouble.

We do not show graphs for the charge exchange polarization and the elastic spin-rotation parameters since our results are very similar to the absorption model calculations of Carerras and White (1970). It is well known that an absorptive type solution with nonsense choosing ρ finds it difficult to give sufficient charge exchange polarization; a ρ which chooses sense works much better. In our solution the average value over the small $|t|$ region ($0 < |t| < 0{\cdot}2$) was about 6%, compared with 16% in the data. (These remarks apply at 6 GeV/c.) At larger $|t|$ our polarization crosses the axis and there is a large negative 'spike' at $|t| = 0{\cdot}5$. This conflicts with the recent data of Bonamy *et al* (1971) showing a positive polarization of about 60% in this region. The sign of the spike can be reversed by a sufficiently large negative value of N, but a study by Halzen and Michael (1971) shows that an absorptive cut model is inconsistent with these data.

Again, the elastic spin rotation parameters (de Lesquen *et al* 1971) are very sensitive to p and f helicity amplitudes. The value of the A parameter in our model for π^-p scattering is close to 100% throughout the $|t|$ range, in agreement with the data (de Lesquen *et al* 1971). The R parameter has a small positive value, whereas the data have a small negative value. The sign can be changed by introducing p and f flip amplitudes to give a negative value for N.

5. Conclusions

To summarize, our conclusions are:

(i) The model can give a good fit to the high energy total cross sections *or* the phase of the forward amplitudes, but not to both quantities simultaneously. The elastic differential cross section data tend to support a solution of the first type.

(ii) Because of the large slope chosen for the pomeron trajectory, and the consequent large real part for the pomeron amplitude, the model cannot explain the elastic polarization data.

(iii) The model is, for charge exchange, very similar to the absorption model, and suffers from the difficulties of that model. p and f flip amplitudes are required to fit the

data well, but there is then a conflict with finite energy sum rules, and the polarization is still a problem.

Thus we see that multiple-pomeron cuts generated by the eikonal model provide a possible explanation for Serpukhov cross section data, but with some undesirable features. It remains for us to comment briefly on some other approaches.

(*a*) The Michigan group of workers (eg Henyey *et al* 1969) have studied inelastic processes by applying the absorption model but multiplying the strength of the cut terms by an arbitrary factor λ, which is greater than unity. This has various desirable effects, in particular moving the cross-over points to lower $|t|$ values. The general conclusion from extensive work on inelastic processes (see the summary by Phillips 1971) is that cuts in nonflip amplitudes need to be enhanced in this way, whereas cuts in flip amplitudes are weak. It is interesting to speculate whether similar enhancement of the cuts generated by multiple-pomeron exchanges can help us; for example, by increasing the effect of the cuts can we fit the Serpukhov σ_{tot} data with a smaller value of α' and so solve the elastic scattering polarization problem? The difficulty is that there are a whole series of cuts, and it is not easy to know how to treat them all. If we multiply the two-p exchange by a factor λ, perhaps the obvious step is to multiply the n-p exchange by λ^{n-1}. A glance at the FM amplitude, equation (8), will show that this is equivalent to reducing α', and we saw in § 4.1 that this makes it *more* difficult to fit the σ_{tot} data.

This is the reverse of what we expect if we examine the first two terms alone, since reducing α' increases the strength of the first cut. However, the amplitude is a series of terms of alternating sign, and this means that even in the low $|t|$ region where the first two terms are the dominant ones, the behaviour of the sum of the series when changes are made in the parameters is sensitive to the effect of those changes on the higher terms. In the absence of any model for treating all the cuts, we conclude that cut enhancement is not a fruitful procedure.

(*b*) Various explanations for the Serpukhov total cross sections, other than vacuum cuts, have been put forward (see the review by Barger and Phillips 1971) though they have not been investigated in such great detail. They only differ decisively at higher energies than those studied so far. The preliminary results from the CERN intersecting storage rings (Holder 1971) show that the rate of shrinkage of the pp elastic differential cross section slows down at these very high energies. It has been suggested (Phillips 1971) that this argues against the eikonal model, since destructive interference between the p and the p–p cut would give more rapid shrinkage. But again it is dangerous to argue from only two terms of the series. We saw in figure 1 that at very low $|t|$ the effective value of α in the FM model is very close to the pole value, and this is still true at intersecting storage ring energies.

Acknowledgments

I would like to thank Dr T J Weare for a helpful discussion and correspondence, and Dr F W D Woodhams for advice on computer programs for least squares fitting.

References

Allaby J V *et al* 1968 *Phys. Lett.* **27B** 49–53
Allaby J V *et al* 1969 *Phys. Lett.* **30B** 500–5

Anderson R *et al* 1970 *Phys. Rev.* D **1** 27–47
Arnold R C and Blackmon M L 1968 *Phys. Rev.* **176** 2082–5
Barger V and Cline D 1970 *Phys. Rev. Lett.* **24** 1313–7
Barger V and Phillips R J N 1968 *Phys. Lett.* **26B** 730–2
—— 1970 *Phys. Rev. Lett.* **24** 291–3
—— 1971 *Nucl. Phys.* B **32** 93–118
Beznogikh G G *et al* 1969 *Phys. Lett.* **30B** 274–5
Bonamy P *et al* 1971 *Proc. Int. Conf. on Elementary Particles, Amsterdam* (Saclay preprint)
Borghini M *et al* 1970 *Phys. Lett.* **31B** 405–9
Branson D 1969 *Phys. Rev.* **179** 1608–16
Carreras B and White J N J 1970 *Nucl. Phys.* B **24** 61–76
Chiu C B and Finkelstein J 1968 *Nuovo Cim.* A **57** 649–64
Cicuta G M and Sugar R L 1971 *Phys. Rev.* D **3** 970–80
Foley K J *et al* 1963 *Phys. Rev. Lett.* **11** 425–9
Foley K J *et al* 1967a *Phys. Rev. Lett.* **19** 857–9
—— 1967b *Phys. Rev. Lett.* **19** 330–2
—— 1969 *Phys. Rev.* **181** 1775–93
Frautschi S and Margolis B 1968 *Nuovo Cim.* A **56** 1155–63
Galbraith W *et al* 1965 *Phys. Rev.* **138** B913–20
Halzen F and Michael C 1971 *Phys. Lett.* **36B** 367–9
Hamer C J and Ravndal F 1970 *Phys. Rev.* D **2** 2687–92
Harting D *et al* 1965 *Nuovo Cim.* **38** 60–94
Henyey F, Kane G L, Pumplin J and Ross M H 1969 *Phys. Rev.* **182** 1579–94
Holder M 1971 *Phys. Lett.* **35B** 355–8
Kaplan J M and Schiff D 1970 *Lett. Nuovo Cim.* **3** 19–22
Kolbig K S and Margolis B 1970 *Phys. Lett.* **31B** 20–1
de Lesquen A *et al* 1971 *Proc. Int. Conf. on Elementary Particles, Amsterdam* (Saclay preprint)
Phillips R J N 1971 *Rutherford High Energy Laboratory Report* no RPP/C/24
Rarita W, Riddell R J, Chiu C B and Phillips R J N 1968 *Phys. Rev.* **165** 1615–39
Sonderegger P *et al* 1966 *Phys. Lett.* **20** 75–8
Stirling A V *et al* 1965 *Phys. Rev. Lett.* **14** 763–7
Vasiljev N M 1970 *Proc. 15th Int. Conf. on High Energy Physics, Kiev* to be published
Weare T J 1970 *Lett. Nuovo Cim.* **4** 251–5
White J N J 1969 *Nucl. Phys.* B **13** 139–52